

# Dynamic Behaviour of Rigid Pavement Plates Due to Loads Moving Diagonally at Constant Speed

Darwis Richarlim<sup>1</sup>, Sofia W. Alisjahbana<sup>2</sup>Faculty of Engineering, Department of Civil Engineering, Tarumanagara University, Jakarta, Indonesia<sup>1</sup>Bakrie University, Jakarta, Indonesia<sup>2</sup>

**ABSTRACT:** Transportation facilities and infrastructure have a very important role in human life. Because the road is an infrastructure that is used every day, it would be better if the highway is properly designed to provide security and comfort for its users. The road that we are going through, is a part of structure called plate. This study will discuss the dynamic behavior of rigid pavement plates supported by the Pasternak Foundation due to the load of vehicles moving diagonally. The rigid pavement modelled as orthotropic plate with semi-rigid boundary conditions on each side of the plate. Modified Bolotin Method is a method used in solving plate problems with semi-rigid boundary conditions. In this study, there are 3 types of soil conditions used, soft soil, medium soil, and hard soil with different values of stiffness and shear modulus. The transverse load across the surface of the plate is modeled as an equivalent single-axis centered load which moves at a constant speed. The analysis in this study is only reviewed where the load is still on the plate ( $0 < t < t_0$ ). Dynamic behavior on plates depend by several factors, such as soil stiffness modulus, Pasternak Foundation shear modulus, load speed, and load angular velocity. Dynamic behavior of the plate that will be obtained are the maximum deflection, shear force and bending moment.

**KEYWORDS:** Rigid pavement, Pasternak Foundation, semi rigid, transversal load, dynamic behaviour.

## I. INTRODUCTION

Transportation facilities and infrastructure have a very important role in human life. In everyday life, we usually use the road as an infrastructure of mobility to where we are headed. Inevitably, damaged roads are still common, because roads are infrastructure that are used every day, so it would be better if the road is designed to truly provide security and convenience for its users. Plate structures for highways that we encounter are generally directly in contact with the ground, the term for this plate is slab on ground or slab on grade. Road pavement can be divided into two types according to the binding material, namely Rigid Pavement and Flexible Pavement. In accordance with its function, namely as a place for mobilizing vehicles, the structure of the highway will be crossed by a moving vehicle (dynamic load). The dynamic response of the plate is influenced by several factors, such as: subgrade conditions, plate thickness and size, vehicle load, vehicle speed.

The development of the theory of plate structures located on supporting soils continues to grow today. Winkler [1] proposed the concept of elastic spring to present the continuous support behaviour of the subgrade under the elastic plate. According to the concept, the vertical stress at each point in the plane of the plate contact with the supporting ground is directly proportional to the deflection of the plate at that point. Pasternak's foundation was stated in 1954, Pasternak's foundation was the development of the true foundation of Winkler. Pasternak considers that the foundation model must also be able to present the ability to hold the shear possessed by the supporting soil. For this reason, Pasternak [2] added a shear layer to the Winkler foundation model, this shear layer is assumed to consist of incompressible vertical elements which function to hold shear.

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: [www.ijirset.com](http://www.ijirset.com)

Vol. 8, Issue 3, March 2019

To determine the natural vibrational frequency of the plate structure with any type of support on its side, it has been started since 1960 by a Russian scientist named Bolotin [3]. The Bolotin method or often referred to as "the edge effect" can solve plate dynamics problems with a combination of various types of plate edge supports. Furthermore, the method was developed by Vijayakumar [4] through his writing entitled "A New Method for Analysis of Flexural Vibration of Rectangular Orthotropic Plates". Later in 2000, the Modified Bolotin Method concept was developed, this method was used to solve numerically plate and shell problems using trigonometric functions. Alisjahbana S.W. [5] conducted a study of simple plates mounted on all four sides and placed on Winkler's foundation. Then in 2001, Alisjahbana S.W.[6] examine the dynamic behavior and stability of a rectangular orthotropic plate with a simple support placed on the Winkler foundation model and loaded with in-plane loads. In 2006, Alisjahbana S.W. and W. Wangsadinata [7] conducted research on square plates Orthotropic lengths supported by Pasternak elastic foundations and burdened by dynamic transverse loads that move at constant speed. In 2018, Halim [8] conducted a study of the dynamic behaviour of rigid pavement plates due to the load moving at a non-constant speed.

## II. LITERATURE REVIEW

### Plate theory

According to Szilard [9], A plate is a structure in the form of a three-dimensional object with a flat and straight surface, and has a thickness that is smaller than the other two dimensions (width and length).Based on the geometrical properties of the structure, generally the plates can be classified as follows:

1. Rigid plate
2. Membrane
3. Flexible plate
4. Thick plate

Based on properties of the material, the plate structure can be classified as follows:

1. Isotropic plate
2. Anisotropic plates
3. Orthotropic plate

### Elasticity Theory

Elasticity theory is a theory that examines the relationship between force, displacement, stress and strain for objects that are linear elastic. Elasticity is the nature of objects that undergo deformation or deformation that is not permanent (returning to the initial condition). If the external force works on a certain material, then the material will change form (deformation) which results in stress and strain. Hooke's law states that the relationship between linear stresses and strains can be expressed in the form of equation as follows:  $\sigma = E \cdot \varepsilon$

### Dynamic Structure Theory

In a structure, there is usually a degree of freedom (Degree of Freedom), which is needed to state the position of a system at any time. A system structure can consist of one degree of freedom (Single Degree of Freedom) and several degrees of freedom (Multi Degree of Freedom) Mathematical models in structural dynamics have several parameters, namely: mass (m), stiffness (k), damping (c), and external forces acting on a structural system with a time function (F(t)).The equation of motion for structural dynamics is based on Newton's Law and the D'Alembert Principle. Newton II law explains that the acceleration of an object will be proportional to the number of forces (resultant force) acting on the object and inversely proportional to its mass, while the D'Alembert principle states that a system can be made in a dynamic equilibrium state by adding a fictional force to external forces called inertial forces [10].

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: [www.ijirset.com](http://www.ijirset.com)

Vol. 8, Issue 3, March 2019

## III.METHODOLOGY

This study begins with the analysis of free vibration with boundary conditions in the form of semi-rigid on the sides of the plate to determine the natural frequency and variety of vibration of the plate, the plate modeled on ground (Pasternak Foundation) which has spring stiffness and shear modulus. Then analysis of force vibration (force vibration) with transverse load in the form of a vehicle that moves diagonally with constant speed to find out how the dynamic response of the plate due to the load. The model of the plate in this study can be seen in Figure 1.

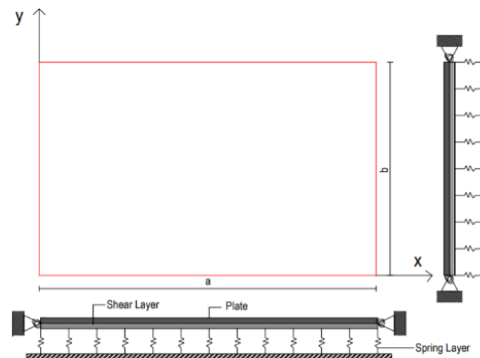


Fig 1: Rigid pavement plates model supported with Pasternak Foundation

### General Equation

The equation of motion [11] of a thin, damped orthotropic rectangular plate loaded by a dynamic transverse load  $p(x,y,t)$  that moves at a constant speed and supported by the foundation Pasternak has a differential equation in general as follows:

$$D_x \left( \frac{\partial^4 w(x,y,t)}{\partial x^4} \right) + 2B \left( \frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} \right) + D_y \left( \frac{\partial^4 w(x,y,t)}{\partial y^4} \right) + \gamma h \frac{\partial w(x,y,t)}{\partial t} + \rho h \left( \frac{\partial^2 w(x,y,t)}{\partial t^2} \right) + k_f w - G_s \left[ \frac{\partial^2 w(x,y,t)}{\partial x^2} + \frac{\partial^2 w(x,y,t)}{\partial y^2} \right] = p(x,y,t)$$

Where:

- |         |   |         |                                  |
|---------|---|---------|----------------------------------|
| $D_x$   | = Flexural rigidity (x axis)              | $D_y$   | = Flexural rigidity (y axis)     |
| $B$     | = Torsional Rigidity                      | $\rho$  | = Density                        |
| $E_x$   | = Modulus of elasticity (x axis)          | $E_y$   | = Modulus of elasticity (y axis) |
| $\nu_x$ | = Poisson's ratio (x axis)                | $\nu_y$ | = Poisson's ratio (y axis)       |
| $k_f$   | = Spring constant of Pasternak Foundation |         |                                  |
| $G_s$   | = Shear modulus of Pasternak Foundation   |         |                                  |
| $h$     | = Plate thickness                         |         |                                  |

$w(x,y,t)$  = Deflection function (depends on the position and time functions)

$p(x,y,t)$  = Load function (depends on the position and time functions)

### Free Vibration Analysis

Assuming that the condition of the edge of the plate is simply supported where the deflection and moment occurring at the edge of the plate are zero, the total system solution can be expressed in the following equation:

$$W_{mn}(x,y) = \left[ A_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \right] T(t)$$

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: [www.ijirset.com](http://www.ijirset.com)

Vol. 8, Issue 3, March 2019

The natural frequency of the system can be obtained from undamped equations as stated as:

$$(\omega_{mn})^2 = \frac{1}{\rho h} \left[ D_x \left( \frac{m\pi}{a} \right)^4 + 2B \left( \frac{m\pi}{a} \right)^2 \left( \frac{n\pi}{b} \right)^2 + k_f + D_y \left( \frac{n\pi}{b} \right)^4 + G_s \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] \right]$$

### First Auxiliary Equation

To obtain the function form in the direction of x, the system spatial solution is assumed to be as follows:

$$W(x, y) = X(x) \sin \left[ \frac{q\pi y}{b} \right]$$

$$X(x) = A_1 \cosh \left( \frac{\beta\pi x}{ab} \right) + A_2 \sinh \left( \frac{\beta\pi x}{ab} \right) + A_3 \cos \left( \frac{p\pi x}{a} \right) + A_4 \sin \left( \frac{p\pi x}{a} \right)$$

To be able to complete the above equation, a boundary condition is needed on the side of the plate. In this study, it is assumed that the plate side has semi-rigid boundary conditions. The boundary conditions of orthotropic rectangular plates that have asymmetrical placement and are semi-rigid can be expressed by the following equation:

$$\begin{aligned} -D_x \left( \frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} \right) &= k_1 \frac{\partial w}{\partial x} & \text{and} & & w(x, y) &= 0 \text{ at } x = 0 \\ -D_x \left( \frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} \right) &= k_1 \frac{\partial w}{\partial x} & \text{and} & & w(x, y) &= 0 \text{ at } x = a \end{aligned}$$

If the boundary conditions in equation are substituted into equation above, then we can obtained this following equation:

$$X(x) = \cosh \left( \frac{\pi\beta x}{ab} \right) + \frac{b(c_1 k_1 p - C_1 k_1 p + a(F_1 + F_2) s_1)}{k_1 (b p s_1 - s_1 \beta)} \sinh \left( \frac{\pi\beta x}{ab} \right) - \cos \left( \frac{\pi p x}{a} \right) - \frac{ab(F_1 + F_2) s_1 + (c_1 - C_1) k_1 \beta}{k_1 (b p s_1 - s_1 \beta)} \sin \left( \frac{\pi p x}{a} \right)$$

### Second Auxiliary Equation

To obtain the function form in the direction of x, the system spatial solution is assumed to be as follows:

$$W(x, y) = Y(y) \sin \left[ \frac{p\pi x}{a} \right]$$

$$Y(y) = B_1 \cosh \left( \frac{\pi}{ab} \theta y \right) + B_2 \sinh \left( \frac{\pi}{ab} \theta y \right) + B_3 \cos \left( \frac{q\pi y}{b} \right) + B_4 \sin \left( \frac{q\pi y}{b} \right)$$

To be able to complete the above equation, a boundary condition is needed on the side of the plate. In this study, it is assumed that the plate side has semi-rigid boundary conditions. The boundary conditions of orthotropic rectangular plates that have asymmetrical placement and are semi-rigid can be expressed by the following equation:

$$\begin{aligned} -D_y \left( \frac{\partial^2 w}{\partial x^2} + \nu_x \frac{\partial^2 w}{\partial y^2} \right) &= k_2 \frac{\partial w}{\partial y} & \text{and} & & w(x, y) &= 0 \text{ at } y = 0 \\ -D_y \left( \frac{\partial^2 w}{\partial x^2} + \nu_x \frac{\partial^2 w}{\partial y^2} \right) &= k_2 \frac{\partial w}{\partial y} & \text{and} & & w(x, y) &= 0 \text{ at } y = b \end{aligned}$$

If the boundary conditions in equation are substituted into equation above, then we can obtained this following equation:

$$Y(y) = \cosh \left( \frac{\pi\theta y}{ab} \right) + \frac{a(c_2 k_2 q - C_2 k_2 q + b(F_3 + F_4) s_2)}{k_2 (a q s_2 - s_2 \theta)} \sinh \left( \frac{\pi\theta y}{ab} \right) - \cos \left( \frac{\pi q y}{b} \right) - \frac{ab(F_3 + F_4) s_2 + (c_2 - C_2) k_2 \theta}{k_2 (a q s_2 - s_2 \theta)} \sin \left( \frac{\pi q y}{b} \right)$$

### Homogeneous Solution

Because the homogeneous solution of the plate motion equation is a linear differential equation, the superposition of each of the different p and q values is also the solution to the homogeneous equation of the plate motion. Thus a homogeneous solution (wh) orthotropic rectangular plates can be stated as follows:

$$w_h = w(x, y, t) = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} [X_{pq}(x) Y_{pq}(y)] e^{-\xi \omega_{pq} t} (a_{0[pq]} \cos[\omega_D] t + b_{0[pq]} \sin[\omega_D] t)$$

### Particular Solution

The particular solution of differential equations is a combination of spatial solutions and temporal particulars can be stated as follows:

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: [www.ijirset.com](http://www.ijirset.com)

Vol. 8, Issue 3, March 2019

$$w_p = \int_0^t \left[ \frac{P(x, y, t)}{\rho h Q_{pq}} \int_{x=0}^a X_{pq}(x) dx \int_{y=0}^b Y_{pq}(y) dy \left( \frac{e^{-\xi \omega_{pq}(t-\tau)}}{\sqrt{1-\xi^2 \omega_{pq}}} \sin(\sqrt{1-\xi^2} \omega_{pq}(t-\tau)) \right) \right] d\tau$$

### Total Solution

The total system solution is expressed as  $w = w_h + w_p$  can be stated as:

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} [X_{pq}(x) Y_{pq}(y)] \cdot \left[ e^{-\xi \omega_{pq}} \left[ a_{0pq} e^{(i \omega_{pq} \sqrt{1-\xi^2})(t-t_0)} b_{0pq} e^{(-i \omega_{pq} \sqrt{1-\xi^2})(t-t_0)} \right] + e^{-\xi \omega_{pq} t} \int_0^t \frac{P(x, y, t)}{\rho h Q_{pq}} \int_{x=0}^a Y_{pq}(y) dy \left( \frac{e^{-\xi \omega_{pq} t}}{\sqrt{1-\xi^2 \omega_{pq}}} \sin(\sqrt{1-\xi^2} \omega_{pq}(t-\tau)) \right) d\tau \right]$$

## IV. ANALYSIS AND RESULTS

### Research Data

Plate material properties used in the numerical analysis are presented in Table 1, the soil parameters [12] are presented in Table 2, and the transverse loads are presented in Table 3.

Table 1: Material properties

No.	Parameter	Notation	Value	Unit
1.	Plate length (x axis)	a	6	m
2.	Plate width (y axis)	b	4	m
3.	Plate thickness	h	0.20	m
4.	Density	$\rho$	2400	$\frac{kg}{m^3}$
5.	Flexural stiffness (x axis)	$D_x$	$7.41 \times 10^6$	N.m
6.	Flexural stiffness y axis)	$D_y$	$9.27 \times 10^6$	N.m

Table 2: Soil parameters

Notation	Unit	Soil Condition		
		Soft	Medium	Stiff
$k_f$	$\frac{MN}{\frac{m}{m^2}}$	27.2	54.4	108.8
$G_s$	$\frac{MN}{m}$	9.52	19.04	38.08

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: [www.ijirset.com](http://www.ijirset.com)

Vol. 8, Issue 3, March 2019

Table 3: Transverse load

No.	Parameters	Notation	Value	Unit
1.	Vehicle load (centralized)	$P_o$	80000	N
2.	Load frequency	$\omega_b$	150	$\frac{\text{rad}}{\text{sec}}$
3.	Vehicle coefficient	$\alpha$	0.5	
4.	Vehicle speed	$v$	60	$\frac{\text{km}}{\text{hr}}$
6.	Damping ratio	$\zeta$	5	%

## Results

This research will discuss the critical speed from the load, critical frequency from the load, internal forces occurred at the plate (shear force and bending moment). All results in this research will be presented in chart below. The critical speed and frequency from the load presented in figure 2 below. The blue line represented soft soil, orange line represented medium soil, and grey line represented stiff soil.

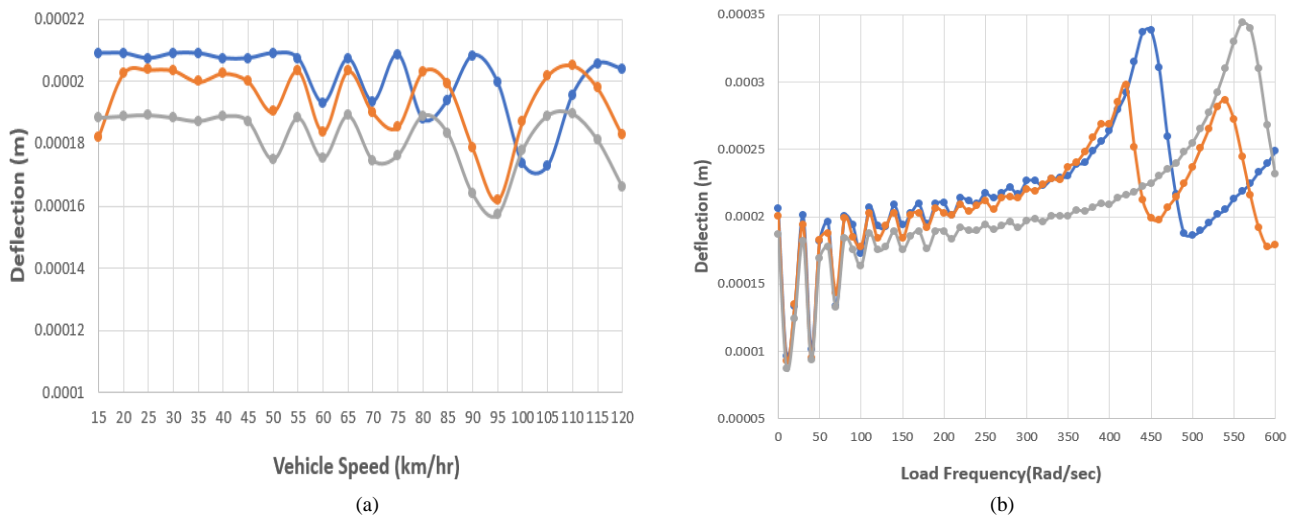


Fig 2: Deflection in different soil condition (a) Critical load speed (b) Critical load frequency

Bending moments and shear forces are derivatives of deflection functions  $w(x, y, t)$ . Bending moment is the second derivative of deflection function and shear force is the third derivative of deflection function. The biggest value of bending moment x direction (15,501.9 N) is produced on the plate which is in the medium soil, the biggest value of bending moment y direction (11,515.05 N) is produced on the plate which is in the soft soil. The biggest value of shear force x direction (32,195.2 N) is produced on the plate that is on stiff soil, the largest value of shear force y direction (30,758.52 N) is produced on the plate which is in soft soil. The shear force and bending moment occurred in plate due to moving load presented in figure 3 and 4 below. The blue line represented soft soil, orange line represented medium soil, and grey line represented stiff soil.

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: [www.ijirset.com](http://www.ijirset.com)

Vol. 8, Issue 3, March 2019

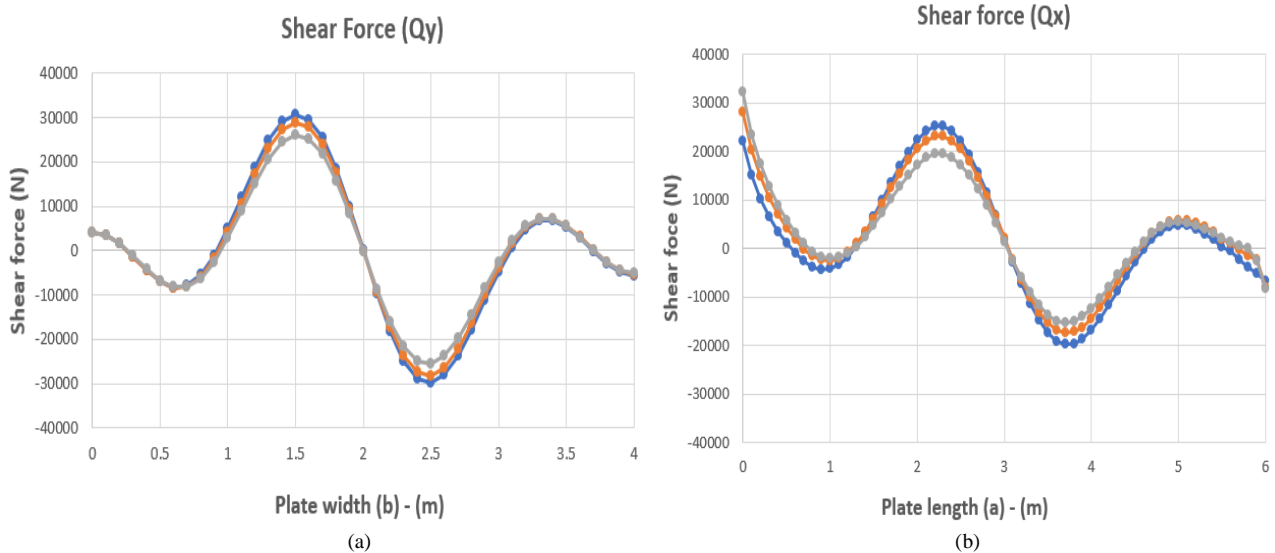


Fig 3: Shear force occurred in plate (a) Shear force y direction (b) Shear force x direction

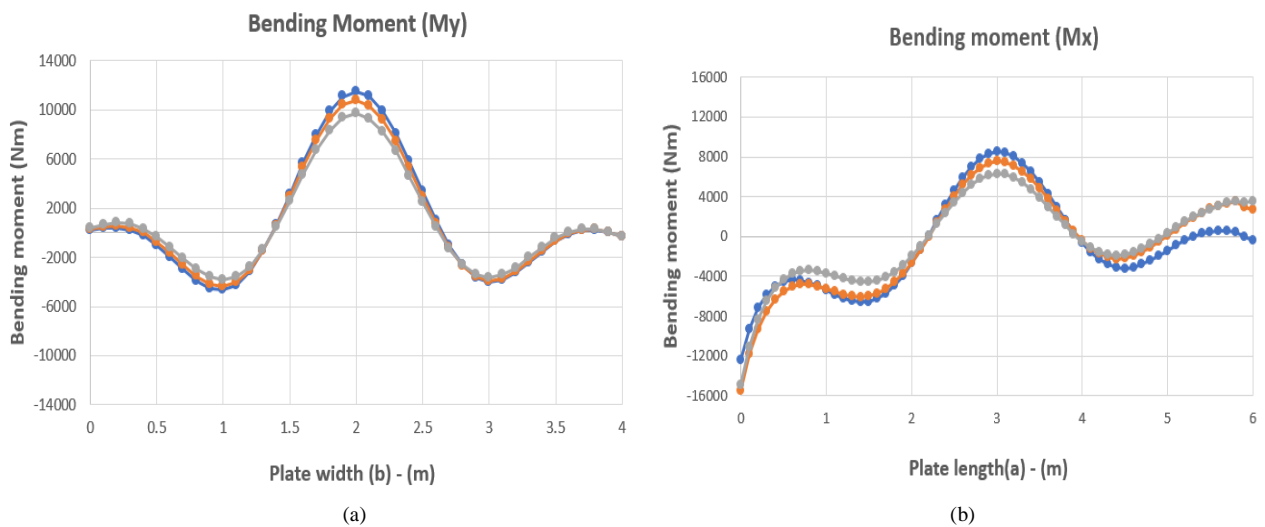


Fig 4: Bending moment occurred in plate (a) Bending moment y direction (b) Bending moment x direction

From the charts above we can observe that the biggest internal force value not always occurred in soft soil. The internal force value depends on the load speed ( $V$ ) and the load angular velocity ( $\omega b$ ) that works.



# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: [www.ijirset.com](http://www.ijirset.com)

Vol. 8, Issue 3, March 2019

## V. CONCLUSION

Based on the results of the analysis carried out in this study, some conclusions can be drawn as follows:

1. The condition of the soil under the plate affects the value of the natural frequency ( $\omega_n$ ) of the plate. Based on the results of the analysis, it was found that the harder the supporting soil conditions (the higher the value of  $k_f$  and  $G_s$ ), the value of the natural frequency ( $\omega_n$ ) of the plate increased.
2. The maximum absolute deflection value of the plate depends on the condition of the soil under the plate. Based on the results of the analysis, it was found that the harder the supporting soil conditions (the higher the value of  $k_f$  and  $G_s$ ), the maximum absolute deflection value on the plate will be smaller.
3. Load motion angle affects the deflection that occurs at the center point of the plate, deflection will decrease if the angle of load moves away from the diagonal direction of the plate.
4. The critical speed of the load is different in the condition of the supporting soil. In this study, the critical velocity in soft soil was 20 km/hr, the soil was 110 km/hr, and on hard soil 108 km/hr.
5. Generally, the deflection value on the plate with the greatest variation in load speed occurs in soft soil and the smallest deflection occurs on hard soil. The maximum deflection value that occurs when the load reaches the critical velocity in soft soil is 0,000209059 m, on medium ground 0,000205258 m, and hard soil 0,000190521 m.
6. The value of the load critical angle velocity ( $\omega_b$ ) with  $V = 60$  km/h varies for various supporting soil conditions. The speed of the critical angle on soft soil is 560 rad/sec, on moderate soil 420 rad/sec, and on hard soil 450 rad/sec.
7. The internal force value depends on the load speed ( $V$ ) and the load angular velocity ( $\omega_b$ ) that works.
8. With loads  $V = 60$  km/h and  $\omega_b = 150$  rad/sec, the value of the largest x direction shear force occurs in hard soil conditions with a value of 32195.2 N, while the largest y direction shear force occurs in soft soil conditions with a value of 30758,52 N.
9. With loads  $V = 60$  km/h and  $\omega_b = 150$  rad/sec, the largest x direction bending moment occurs in moderate soil conditions with a value of 15501.9 Nm, while the greatest bending direction y direction occurs in soft soil conditions with a value of 11515,05 Nm.

## REFERENCES

- [1] E. Winkler, Die Lehre von der Elastizitat und Festigkeit, Dominicus, Prague, 1867.
- [2] Pasternak, P.L. (1954). "A New Method of Analysis of An Elastic Foundation by Means of Two Foundation". Moscow.
- [3] Bolotin, V. V. (1960). "The Edge Effect in The Oscillations of Elastic Shells". PMM, Vol. 24, No. 5, 1960, Moscow, Rusia, pp. 831-843.
- [4] Vijayakumar, K. (1971). "A New Method of Flexural Vibration of Rectangular Orthotropic Plates". Journal of the Aeronautical Society of India, Vol. 23, No. 4, 1971, Indian Institute of Technology.
- [5] Alisjahbana, S. W. (1999). "Respon Dinamik Pelat Persegi Panjang Di Atas Pondasi Winkler Akibat Beban Dinamik Bergerak". Lembaga Penelitian dan Pengembangan Universitas Tarumanagara.
- [6] Alisjahbana, S. W. (2001). "Perilaku Analitis Pelat Orthotropik Akibat Beban In-Plane Bervariasi Secara Linier". Jurusan Teknik Sipil, ISSN: 0853-5272, Juli 2001, Universitas Tarumanagara.
- [7] Alisjahbana, S. W. dan Wangsadinata, W. (2006). "Dynamic Response of Rectangular Orthotropic Plate Supported by Pasternak Foundation Subjected to Dynamic Load". First International Structural Specialty Conference, Calgary, Alberta, Canada.
- [8] Halim, R. (2018). "Perilaku Dinamik Pelat Perkerasan Kaku Akibat Beban Bergerak dengan Kecepatan Tidak Konstan". Tesis Program Magister, Universitas Tarumanagara, Jakarta.
- [9] Szilard, R. (2004). "Theories and Applicatios of Plate Analysis: Classical, Numerical and Engineering Methods". John Wiley & Sons, Inc, Hoboken, New Jersey, USA.
- [10] Paz, Mario., "Structural Dynamics: Theory and Computation Second Edition", Chapman & Hall, New York, 1985.
- [11] Alisjahbana, S.W. (2011). "Dinamika Struktur Pelat II". B Press, Universitas Bakrie, Jakarta.
- [12] Baadilla, Douglas Abdullah.(2006). "Dinamika Pelat Perkerasan Kaku dengan Tahanan Tepi Sembarang". Disertasi Doktor Teknik Sipil, Pascasarjana Universitas Tarumanagara, Jakarta.